

Now we agree that for $f(p) = (2p + 8)^3$ the inverse function is $f^{-1}(q) = (q^{1/3} - 8)/2$.
(and that letters only matter for function consistency)

But in several places, the article does not agree with this correct example of "inverse function".
The problem here, and in many sources, seems to stem from confusing "inverse function"
with other meanings of "inverse".

One error is below the table, where a "formula" for inverse function is incorrect, in that
the result is just a transposed restatement of the function. (some might say inverse)

It states there that if $y = f(x) = (2x + 8)^3$ then $X = (Y^{1/3} - 8)/2$. (as transposed)

Then the error: "Thus the inverse function f^{-1} is $f^{-1}(Y) = (Y^{1/3} - 8)/2$ ".

The two primary tests for inverse functions (subject to domains) are the $X=Y$ reflection,
and $f(f^{-1}) = f^{-1}(f) = X$ or Y (the "unchanged" identity).

For the reflection test, take two points in $f(x)$, maybe $\{(0, 512), (-2, 64)\}$.

An inverse function should have points $\{512, 0\}$ and $\{64, -2\}$. But with the form being tested,
 $Y=0$ yields $f^{-1}(Y) = -4$, and $Y=64$ yields $f^{-1}(Y) = -2$; so it's not an inverse function.

Comparing to the original $f(x)$, $Y=0$ yields $f(x) = -4$ and $Y=64$ yields -2 , showing that the
tested $f^{-1}(y)$ is just the original function rearranged by transposition, not an "inverse function".

The actual "inverse function" in terms of Y is $f^{-1}(y) = (2y + 8)^3$. Here $Y=0$ yields 512,
and $Y = -2$ yields 64, showing reflection across $X=Y$.

Although $f^{-1}(x)$ is easier to think with and plot, composing as $f[f^{-1}(y)]$ gives you identity Y .

And composing as $f^{-1}[f(y)]$ gives you identity Y . Again proving a correct "inverse function".

$X = e^Y$ is sometimes referred to as the "inverse" of $Y = \ln X$, but it's just another form
of the same log function. The actual "inverse function" is $Y = e^X$.

Another confusion of "inverse" is in conversion of temperature scales (as in the article).

Reversing a relationship of F in terms of C does not produce a "function inverse";

it produces a transposed reversal, sometimes referred to as an "inverse relationship".